Santosh Academia

Solutions to JEE (Main) - 2021

Test Date: 31st August 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper- 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the end of light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its midpoint ?

(A) 1.905×10^5 kgm^2 (B) 1.875×10^5 kgm^2

(C) 18.75 kgm^2 (D) 19.05 kgm^2

Q2. If RE be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is :

(A) $1+\frac{r}{R_E}+\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$ (B) $1-\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$

(C) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$ (D) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

Q3. Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to :-

(A) $\sqrt{\frac{m_e}{m_p}}$ (B) $\sqrt{\frac{m_p}{m_e}}$

(C) $\frac{m_p}{m_e}$ (D) $\left(\frac{m_p}{m_e}\right)^{3/2}$

Q4. Two thin metallic spherical shells of radii r1 and r2 (r1 < r2) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature $\theta 1$ and the outer shell at temperature $\theta 2(\theta 1<\theta 2)$. The rate at which heat flows radially through the material is :-

(A) $\frac{\pi r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$ (B) $\frac{K(\theta_2-\theta_1)(r_2-r_1)}{4\pi r_1 r_2}$

(C) $\frac{K(\theta_2-\theta_1)}{r_2-r_1}$ (D) $\frac{4\pi K r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$

Q5. For a body executing S.H.M. :

(a) Potential energy is always equal to its K.E.

(B) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

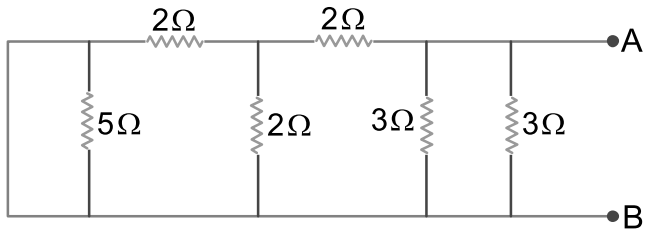
(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below :

(A) only (b) (B) (b) and (c)

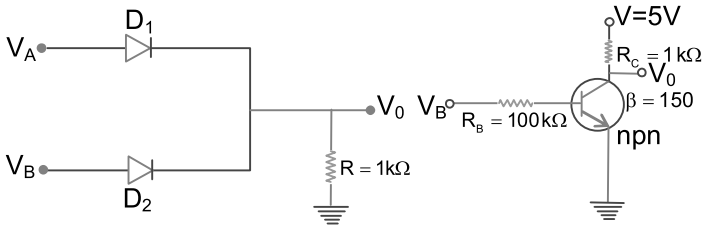
(C) only (c) (D) (c) and (d)

Q6. The equivalent resistance of the given circuit between the terminals A and B is :



(A) 0 Ω
(B) 1 Ω
(C) $\frac{9}{2}\Omega$
(D) 3 Ω

Q7. If VA and VB are the input voltages (either 5V or 0V) and Vo is the output voltage then the two gates represented in the following circuit (A) and (B) are:-



(A) (B)

(A) NAND and NOR Gate
(B) OR and NOT Gate
(C) AND and NOT Gate
(D) AND and OR Gate

Q8. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50 × 10^3 kg, The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column.
[Use Y = 2.0 × 10^{11} Pa, g = 9.8 m/s^2]
(A) 3.60 × 10^{-8} (B) 2.60 × 10^{-7}
(C) 1.87 × 10^{-3} (D) 7.07 × 10^{-4}

Q9. If velocity [V], time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be :
(A) [FT^2 V] (B) [FTV^{-1}]
(C) [FVT^{-1}] (D) [FT^{-1} V^{-1}]

Q10. The magnetic field vector of an electromagnetic wave is given by $B = B_o = \frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz-\omega t)$; where $\hat{i},\hat{j}$ represents unit vector along x and y-axis respectively. At t = 0 s, two electric charges q1 of 4π coulomb and q2 of 2π coulomb located at $\left(0,0,\frac{\pi}{k}\right)$ and $\left(0,0,\frac{3\pi}{k}\right)$, respectively, have the same velocity of 0.5 c $\hat{i}$ (where c is the velocity of light). The ratio of the force acting on charge q1 to q2 is :-
(A) 1: $\sqrt{2}$ (B) $\sqrt{2}$: 1
(C) 2 : 1 (D) $2\sqrt{2}$: 1

Q11. A mixture of hydrogen and oxygen has volume 500 cm^3, temperature 300 K, pressure 400 kPa and mass 0.76 g. The ratio of masses of oxygen to hydrogen will be :-
(A) 3 : 16 (B) 16 : 3
(C) 3 : 8 (D) 8 : 3

Q12. **Statement-1:** If three forces $\vec{F}_1,\vec{F}_2$ and $\vec{F}_3$ are represented by three sides of a triangle and, $\vec{F}_1+\vec{F}_2=-\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.
Statement-II: A triangle made up of three forces $\vec{F}_1,\vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below:
(A) Statement-I is false but Statement-II is true
(B) Both Statement-I and Statement-II are true
(C) Statement-I is true but Statement-II is false
(D) Both Statement-I and Statement-II are false

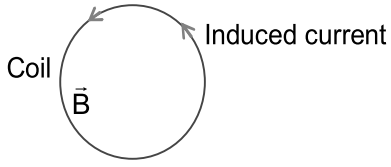
Q13. A free electron of 2.6 eV energy collides with a H^+ ion. This results in the formation of a hydrogen atom in the first excited state and a photon is released. Find the frequency of the emitted photon. ($h = 6.6 \times 10^{-34}$ Js)
(A) 0.19×10^{15} MHz
(B) 1.45×10^{9} MHz
(C) 1.45×10^{16} MHz
(D) 9.0×10^{27} MHz

Q14. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is :-
(A) $\frac{2}{3}$
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$

Q15. **Statement-I:** Two forces $\left(\vec{P}+\vec{Q}\right)$ and $\left(\vec{P}-\vec{Q}\right)$ where $\vec{P} \perp \vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3\left(P^2+Q^2\right)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2\left(P^2+Q^2\right)}$. This is possible only when $\theta_1 < \theta_2$.
Statement-II: In the situation given above.
$\theta_1 = 60°$ and $\theta_2 = 90°$
In the light of the above statements, choose the most appropriate answer from the options given below :-
(A) Both Statement-I and Statement-II are true
(B) Both Statement-I and Statement-II are false
(C) Statement-I is true but Statement-II is false
(D) Statement-I is false but Statement-II is true

Q16. A bob of mass 'm' suspended by a thread of length l undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by 1/3rd of the original length, then the time period of the simple harmonic oscillations will be :-
(A) $\frac{3}{4}$T
(B) $\frac{3}{2}$T
(C) T
(D) $\frac{4}{3}$T

Q17. A coil is placed in a magnetic field as shown below : B
A current is induced in the coil because $\vec{B}$ is:
(A) Outward and decreasing with time
(B) Parallel to the plane of coil and decreasing with time
(C) Parallel to the plane of coil and increasing with time
(D) Outward and increasing with time



Q18. **Statement–I :** To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load RL.
Statement–II : To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with RL.
In the light of the above statements, choose the most appropriate answer from the options given below :
(A) **Statement I** is false but **Statement II** is true
(B) **Statement I** is true but **Statement II** is false
(C) Both **Statement I** and **Statement II** are true
(D) Both **Statement I** and **Statement II** are false

Q19. A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is :
(Assume that the current is flowing in the clockwise direction.)
(A) $2\sqrt{3}\times10^{-7}$ T, outside the plane of triangle
(B) $2\sqrt{3}\times10^{-5}$ T, inside the plane of triangle
(C) 3×10^{-5} T, inside the plane of triangle
(D) 3×10^{-7} T, outside the plane of triangle

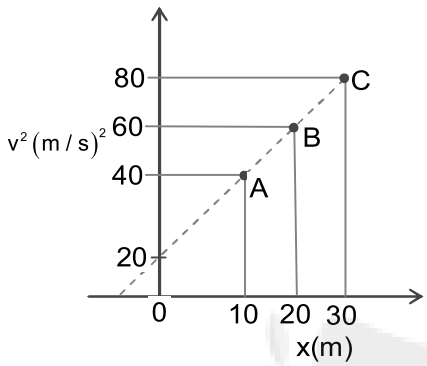
Q20. Choose the incorrect statement :
(a) The electric lines of force entering into a Gaussian surface provide negative flux.
(b) A charge 'q' is placed at the centre of a cube. The flux through all the faces will be the same.
(c) In a uniform electric field net flux through a closed Gaussian surface containing no net charge, is zero.
(d) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.
Choose the most appropriate answer from the options given below
(A) (c) and (d) only
(B) (a) and (c) only
(C) (d) only
(D) (b) and (d) only

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is _____m/s^2.



Q2. The diameter of a spherical bob is measured using a vernier callipers. 9 divisions of the main scale, in the vernier callipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8^{th} division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is_____ $\times 10^{-2}$ cm.

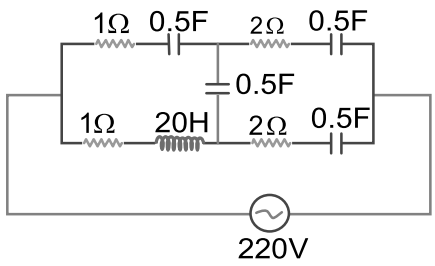
Q3. A resistor dissipates 192 J of energy in 1 s when a current of 4A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5 s in_____J.

Q4. A bandwidth of 6 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this band simultaneously without interfering with each other will be_____.

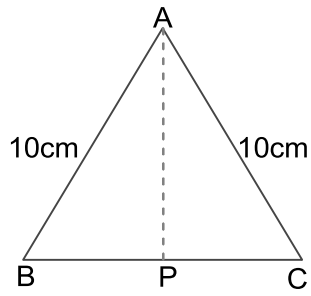
Q5. A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume 10^3 cm^3. If the core material is replaced by another material having relative permeability of 750 with same volume maintaining same current of 0.75 A in the solenoid, the fractional change in the magnetic moment of the core would be approximately $\left(\frac{x}{499}\right)$. Find the value of x.

Q6. In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright is 2.4 cm. The frequency of light used is_____$\times 10^{14}$ Hz.

Q7. At very high frequencies, the effective impendance of the given circuit will be_____Ω.



Q8. Cross–section view of a prism is the equilateral triangle ABC in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (midpoint of BC) to A is_____ $\times 10^{-10}$ s. (Given, speed of light in vacuum = 3×10^8 m/s and $\cos 30^\circ = \frac{\sqrt{3}}{2}$)



Q9. A parallel plate capacitor of capacitance 200 μF is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be_____J.

Q10. A sample of gas with γ = 1.5 is taken through an adiabatic process in which the volume is compressed from 1200 cm^3 to 300 cm^3. If the initial pressure is 200 kPa. The absolute value of the work done by the gas in the process =_____J.

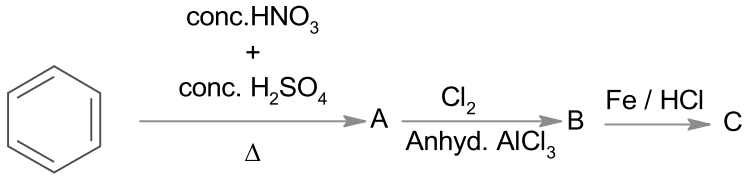
PART – B (CHEMISTRY)

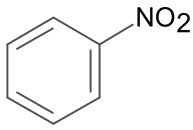
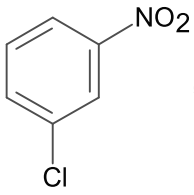
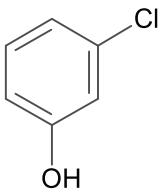
SECTION - A

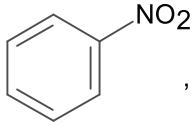
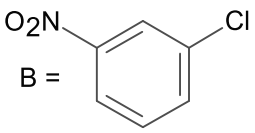
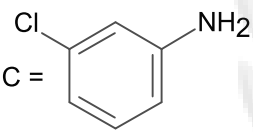
(One Options Correct Type)

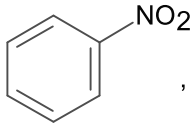
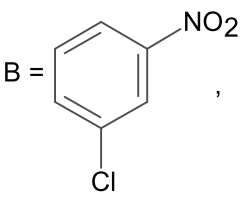
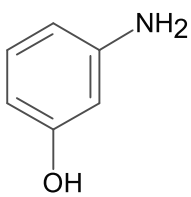
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

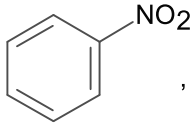
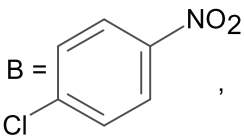
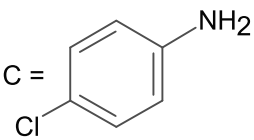
Q1. Identify correct A, B and C in the reaction sequence given below :



(A) A =  , B =  , C = 

(B) A =  ,  , 

(C) A =  ,  , 

(D) A =  ,  , 

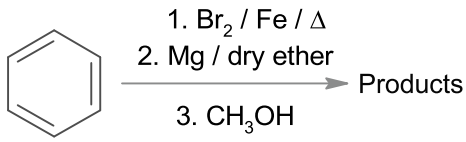
Q2. Match List-I with List-II.

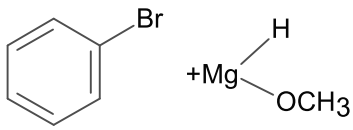
List – I (Parameter)		**List – II (Unit)**	
(a)	Cell constant	(i)	S cm^2 mol^{-1}
(b)	Molar conductivity	(ii)	Dimensionless
(c)	Conductivity	(iii)	m^{-1}
(d)	Degree of dissociation of electrolyte	(iv)	Ω^{-1} m^{-1}

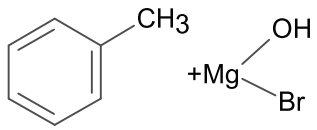
Choose the most appropriate answer from the options given below:

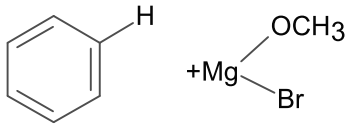
(A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
(B) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
(C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
(D) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

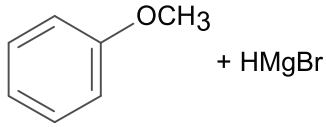
Q3. For following sequence of reactions, the correct products are:



(A) 

(B) 

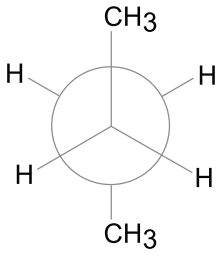
(C) 

(D) 

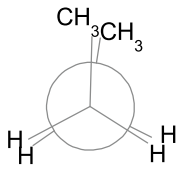
Q4. Which one of the following correctly represents the order of stability of oxides, X_2O; (X = halogen) ?

(A) Cl > I > Br
(B) Br > I > Cl
(C) Br > Cl > I
(D) I > Cl > Br

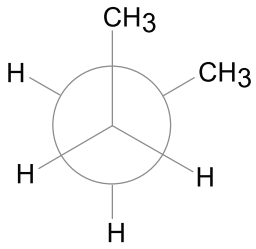
Q5. Arrange the following conformational isomers of n-butane in order of their increasing potential energy :



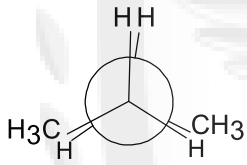
I



II



III



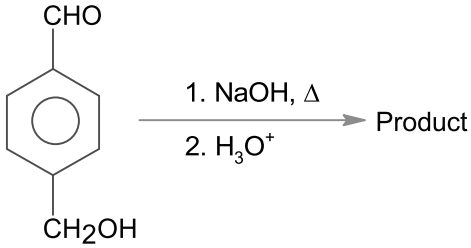
IV

(A) I < III < IV < II
(B) I < IV < III < II
(C) II < III < IV < I
(D) II < IV < III < I

Q6. The Eu^{2+} ion is a strong reducing agent in spite of its ground state electronic configuration (outermost) : [Atomic number of Eu = 63]

(A) $4f^7$
(B) $4f^6 6s^2$
(C) $4f^7 6s^2$
(D) $4f^6$

Q7. For the reaction given below :



The compound which is not formed as a product in the reaction is a :

(A) dicarboxylic acid
(B) diol
(C) compound with both alcohol and acid functional groups
(D) monocarboxylic acid

Q8. Which of the following is NOT an example of fibrous protein ?

(A) Keratin (B) Myosin

(C) Collagen (D) Albumin

Q9. Spin only magnetic moment in BM of $[Fe(CO)_4(C_2O_4)]^+$ is :

(A) 0 (B) 5.92

(C) 1.73 (D) 1

Q10. The **incorrect** expression among the following is:

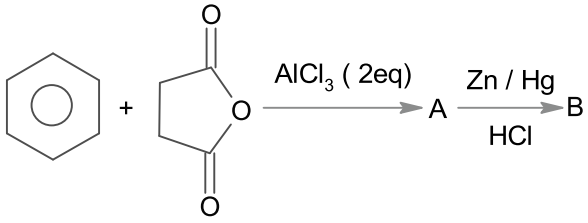
(A) $\ln K = \frac{\Delta H^o - T\Delta S^o}{RT}$

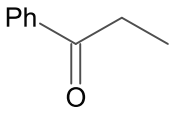
(B) For isothermal process $w_{reversible} = -nRT\ln\frac{V_f}{V_i}$

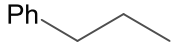
(C) $\frac{\Delta G_{System}}{\Delta S_{Total}} = -$ T (at constant P)

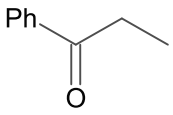
(D) $K = e^{-\Delta G^o/RT}$

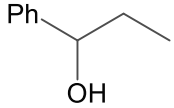
Q11. The structures of A and B formed in the following reaction are : [Ph = – C_6H_5]

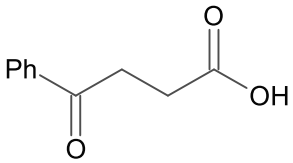


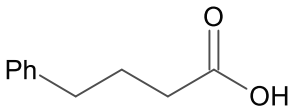
(A) A = 

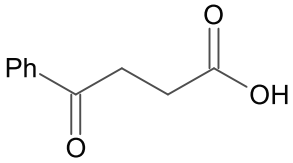
, B = 

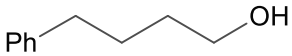
(B) A = 

, B = 

(C) A = 

, B = 

(D) A = 

, B = 

Q12. Which among the following is not a polyester ?

(A) Glyptal (B) Dacron

(C) Novolac (D) PHBV

Q13. Match List-I with List-II.

List – I (Metal ion)		**List – II (Group in Qualitative Analysis)**	
(a)	Mn^{2+}	(i)	Group – III
(b)	As^{3+}	(ii)	Group – IIA
(c)	Cu^{2+}	(iii)	Group – IV
(d)	Al^{3+}	(iv)	Group – IIB

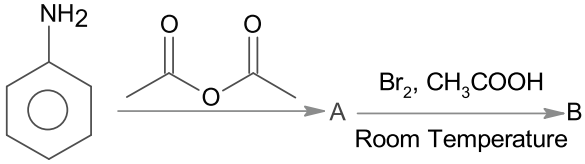
Choose the most appropriate answer from the options given below:

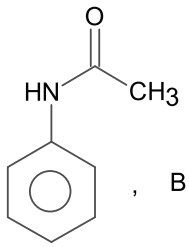
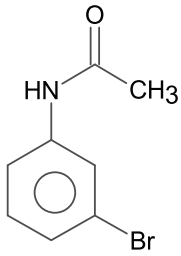
(A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(B) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
(C) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)
(D) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

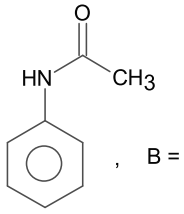
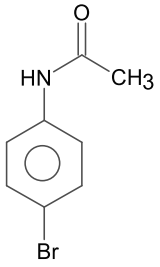
Q14. The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively, X and Y are :

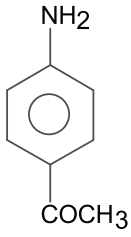
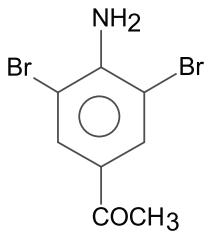
(A) X = Ammonium salts, Y = SO_2
(B) X = SO_2, Y = Ammonium salts
(C) X = Ammonium salts, Y = CO_2
(D) X = CO_2, Y = SO_2

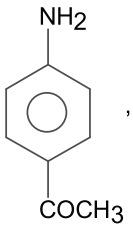
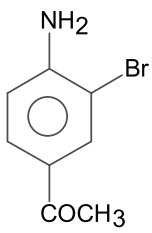
Q15. The major products A and B formed in the following reaction sequence are :



(A) A =  , B = 

(B) A =  , B = 

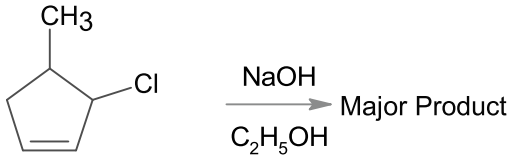
(C) A =  , B = 

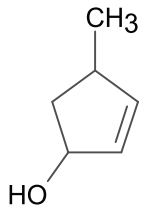
(D) A =  , B = 

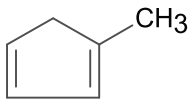
Q16. The number of S=O bonds present in sulphurous acid, peroxodisulphuric acid and pyrosulphuric acid, respectively are :

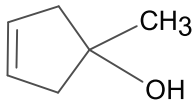
(A) 2, 3 and 4
(B) 1, 4 and 3
(C) 1, 4 and 4
(D) 2, 4 and 3

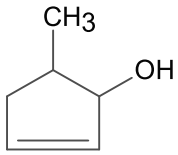
Q17. The major product of the following reaction is:



(A) 

(B) 

(C) 

(D) 

Q18. Which one of the following statements is incorrect ?

(A) Atomic hydrogen is produced when H_2 molecules at a high temperature are irradiated with UV radiation.
(B) Bond dissociation enthalpy of H_2 is highest among diatomic gaseous molecules which contain a single bond.
(C) Dihydrogen is produced on reacting zinc with HCl as well as $NaOH_{(aq)}$.
(D) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%.

Q19. In which one of the following sets all species show disproportionation reaction ?

(A) MnO_4^-, ClO_2^-, Cl_2 and Mn^{3+}
(B) ClO_2^-, F_2, MnO_4^- and $Cr_2O_7^{2-}$
(C) $ClO_4^-, MnO_4^-, ClO_2^-$ and F_2
(D) $Cr_2O_7^{2-}, MnO_4^-, ClO_2^-$ and Cl_2

Q20. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Lithium salts are hydrated.
Reason (R) : Lithium has higher polarising power than other alkali metal group members.
In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(B) (A) is correct but (R) is not correct .
(C) (A) is not correct but (R) is correct.
(D) Both (A) and (R) are correct but (R) is NOT the correct explanation of (A).

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The transformation occurring in Duma's method is given below :

$$C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$$

The value of *y* is _____. (Integer answer)

Q2. CH_4 is adsorbed on 1 g charcoal at 0°C following the Freundlich adsorption isotherm. 10.0 mL of CH_4 is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of CH_4 adsorbed at 300 mm of Hg is 10^x mL. The value of x is _____ × 10^{-2}.
(Nearest integer)
[Use $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

Q3. According to molecular orbital theory, the number of unpaired electron(s) in O_2^{2-} is:

Q4. The pH of a solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is $x \times 10^{-4}$. The value of x is _____. (Nearest integer)
[log 2.5 = 0.3979]

Q5. The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites is A_xB. The value of x is _____.
(Integer answer)

Q6. 1.22 g of an organic acid is separately dissolved in 100 g of benzene (K_b = 2.6 K kg mol^{-1}) and 100 g of acetone (K_b = 1.7 K kg mol^{-1}). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C. The increase in boiling point of solution in benzene in °C is x × 10^{-2}. The value of x is _____.(Nearest integer)
[Atomic mass : C = 12.0, H = 1.0, O= 16.0]

Q7. For the reaction A → B, the rate constant K(in s^{-1}) is given by

$$\log_{10} K = 20.35 - \frac{\left(2.47 \times 10^3\right)}{T}.$$

The energy of activation in kJ mol^{-1} is _____. (Nearest integer)
[Given : R = 8.314 J K^{-1} mol^{-1}]

Q8. The value of magnetic quantum number of the outermost electron of Zn^+ ion is _____.

Q9. Sodium oxide reacts with water to produce sodium hydroxide. 20.0 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is _____ × 10^{-1} M. (Nearest integer)
[Atomic mass : Na = 23.0, O = 16.0, H = 1.0]

Q10. In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is _____.
Pb, Sb, Se, Te, Ru, Ag, Au and Pt

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The domain of the function $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ is :

(A) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$ (B) $\left[0,\frac{1}{2}\right]$

(C) $\left[0,\frac{1}{4}\right]$ (D) $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$

Q2. If $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x>0, \phi>0$, and $y(1)=-1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to :

(A) $2\phi(1)$ (B) $4\phi(2)$

(C) $4\phi(1)$ (D) $\phi(1)$

Q3. Let f : N $\to$ N be a function such that f(m + n) = f(m) + f(n) for every m, n $\in$ N. If f(6) = 18, then $f(2).f(3)$ is equal to :

(A) 18 (B) 36

(C) 54 (D) 6

Q4. The number of solutions of the equation $32^{\tan^2 x}+32^{\sec^2 x}=81, 0\le x\le\frac{\pi}{4}$ is:

(A) 0 (B) 2

(C) 1 (D) 3

Q5. If $\alpha=\lim\limits_{x\to\frac{\pi}{4}}\frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ and $\beta=\lim\limits_{x\to 0}(\cos x)^{\cot x}$ are the roots of the equation, $ax^2+bx-4=0$, then the ordered pair (a, b) is:

(A) (1, 3) (B) (–1, –3)

(C) (1, –3) (D) (–1, 3)

Q6. Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies $\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{b})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$,

then $\vec{r}$ is equal to

(A) $\frac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$ (B) $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{c}\right)$

(C) $\frac{1}{3}\left(2\vec{a}+\vec{b}-\vec{c}\right)$ (D) $\frac{1}{2}\left(\vec{a}+\vec{b}+2\vec{c}\right)$

Q7. Let A be the set of all points (α,β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α,β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

(A) $\frac{16}{\sqrt{5}}$ (B) $\frac{4}{\sqrt{5}}$

(C) $\frac{12}{\sqrt{5}}$ (D) $\frac{8}{\sqrt{5}}$

Q8. The distance of the point (–1, 2, –2) from the line of intersection of the planes 2x + 3y + 2z = 0 and x – 2y + z = 0 is :

(A) $\frac{5}{2}$ (B) $\frac{\sqrt{42}}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{34}}{2}$

Q9. An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab, a>b$ is:

(A) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (B) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

(C) $\tan^{-1}\left(2\sqrt{ab}\right)$ (D) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

Q10. Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is :

(A) $\frac{1}{15}$ (B) $\frac{1}{5}$

(C) $\frac{1}{10}$ (D) $\frac{1}{30}$

Q11. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is:

(A) $\sim p \wedge q \wedge \sim r$ (B) $\sim p \wedge q \wedge r$

(C) $p \wedge q \wedge r$ (D) $p \wedge \sim q \wedge \sim r$

Q12. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is:

(A) $6\sqrt{2}$ (B) $2\sqrt{2}$

(C) $2\sqrt{2}-1$ (D) $3\sqrt{2}$

Q13. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations
$x + (\cos\gamma)y + (\cos\beta)z = 0$
$(\cos\gamma)x + y + (\cos\alpha)z = 0$
$(\cos\beta)x + (\cos\alpha)y + z = 0$
has :
(A) infinitely many solutions
(B) exactly two solutions
(C) no solution
(D) a unique solution

Q14. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}, p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

(A) $\frac{21}{19}$
(B) $\frac{100}{121}$
(C) $\frac{121}{100}$
(D) $\frac{19}{21}$

Q15. The sum of the roots of the equation, $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is:
(A) $\log_2 12$
(B) $\log_2 11$
(C) $\log_2 13$
(D) $\log_2 14$

Q16. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) = 0, f(1) = 1 and f(2) = 2, then
(A) f''(x) = 0 for all $x \in (0, 2)$
(B) f''(x) > 0 for all $x \in (0, 2)$
(C) f'(x) = 0 for some $x \in [0, 2]$
(D) f''(x) = 0 for some $x \in (0, 2)$

Q17. If [x] is the greatest integer $\leq x$, then $\pi^2 \int_0^2 \left(\sin\frac{\pi x}{2}\right)\left(x - [x]\right)^{[x]} dx$ is equal to:

(A) $4(\pi + 1)$
(B) $4(\pi - 1)$
(C) $2(\pi + 1)$
(D) $2(\pi - 1)$

Q18. If $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y}\log_e 2}, y(0) = 0$, then for y = 1, the value of x lies in the interval:

(A) (1, 2)
(B) $\left(\frac{1}{2}, 1\right]$
(C) $\left(0, \frac{1}{2}\right]$
(D) (2, 3)

Q19. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

(A) $\frac{92}{5}$
(B) $\frac{134}{5}$
(C) $\frac{536}{25}$
(D) $\frac{112}{5}$

Q20. The locus of mid.points of the line segments joining (–3, –5) and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:
(A) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$
(B) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$
(C) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$
(D) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x + 3y – 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to_____.

Q2. Let f(x) be a cubic polynomial with f(1) = –10, f(–1) = 6, and has a local minima at x = 1, and f'(x) has a local minima at x = –1. Then f(3) is equal to _____.

Q3. If $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx=\alpha\log_e\left|1+\tan x\right|+\beta\log_e\left|1-\tan x+\tan^2 x\right|+\gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$, when C is constant of integration, then the value of $18\left(\alpha+\beta+\gamma^2\right)$ is _____.

Q4. If the line y = mx bisects the area enclosed by the lines x = 0, y = 0, $x = \frac{3}{2}$ and the curve $y = 1 + 4x – x^2$, then 12 m is equal to _____.

Q5. A tangent line L is drawn at the point (2, –4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to_____.

Q6. Let B be the centre of the circle $x^2 + y^2 – 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8\cdot\left(\frac{\text{area }\Delta APQ}{\text{area }\Delta BPQ}\right)$ is equal to _____.

Q7. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is ____.

Q8. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K\cdot 2^{16}$, then K is equal to _____.

Q9. If $S=\frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+.....$, then 160 S is equal to _____.

Q10. The number of elements in the set $\left\{A=\begin{pmatrix}a & b\\ 0 & d\end{pmatrix}: a,b,d\in\{-1,0,1\}\text{ and}(1-A)^3=I-A^3\right\}$, where I is 2 × 2 identify matrix, is _____.

Santosh Academia

Keys to JEE (Main) - 2021

PART – A (PHYSICS)

SECTION - A

1.	**D**	2.	**C**	3.	**B**	4.	**D**
5.	**D**	6.	**B**	7.	**B**	8.	**B**
9.	**B**	10.	**C**	11.	**B**	12.	**B**
13.	**B**	14.	**D**	15.	**A**	16.	**D**
17.	**A**	18.	**C**	19.	**C**	20.	**C**

SECTION - B

1.	**1**	2.	**52**	3.	**3840**	4.	**500**
5.	**250**	6.	**5**	7.	**2**	8.	**5**
9.	**4**	10.	**480**				

PART – B (CHEMISTRY)

SECTION - A

1.	**B**	2.	**C**	3.	**C**	4.	**D**
5.	**A**	6.	**A**	7.	**A**	8.	**D**
9.	**C**	10.	**A**	11.	**C**	12.	**C**
13.	**B**	14.	**A**	15.	**B**	16.	**C**
17.	**D**	18.	**D**	19.	**A**	20.	**D**

SECTION - B

1.	**7**	2.	**128**	3.	**0**	4.	**6021**
5.	**1**	6.	**13**	7.	**47**	8.	**0**
9.	**13**	10.	**6**				

PART – C (MATHEMATICS)

SECTION - A

1.	**A**	2.	**C**	3.	**C**	4.	**C**
5.	**A**	6.	**A**	7.	**D**	8.	**D**
9.	**B**	10.	**C**	11.	**D**	12.	**B**
13.	**A**	14.	**D**	15.	**B**	16.	**D**
17.	**B**	18.	**A**	19.	**C**	20.	**A**

SECTION – B

1.	**7**	2.	**22**	3.	**3**	4.	**26**
5.	**2**	6.	**18**	7.	**5143**	8.	**315**
9.	**305**	10.	**8**				

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Solutions to JEE (Main) - 2021

PART – A (PHYSIC S)

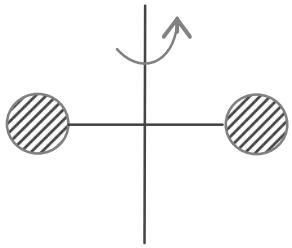
SECTION - A

Sol1. Using parallel axis theorem:

$I=2\times\left[I_{cm}+md^2\right]$

$I=2\times[\frac{2}{5}(1.5)(0.5)^2+(1.5)(2.5)^2]$

$I=19.05\ kgm^2$



Sol2. Acceleration due to gravity at r distance above the surface= $\frac{GM}{(R+r)^2}$

Acceleration due to gravity at r distance below the surface= $\frac{GM}{R^3}(R-r)$

So, ratio = $\frac{(R-r)(R+r)^2}{R^3}=\frac{(R-r)(R^2+r^2+2Rr)}{R^3}=1+\frac{r}{R}-\frac{r^2}{R^2}-\frac{r^3}{R^3}$

Sol3. Heisenberg uncertainty principle:

$\Rightarrow\Delta x\Delta p\geq\hbar$

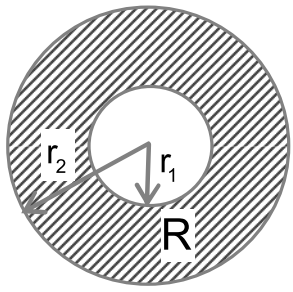
$\Rightarrow\Delta x(m\Delta v)\geq\hbar$

$\Rightarrow\Delta x(m\sqrt{\frac{3RT}{m}})\geq\hbar$

$\Rightarrow\Delta x\propto\frac{1}{\sqrt{m}}$

Sol4. Thermal resistance of spherical shell= $\frac{r_2-r_1}{4\pi Kr_1r_2}$

Rate of heat flow= $\frac{\Delta T}{R}=\frac{(\theta_2-\theta_1)}{(\frac{r_2-r_1}{4\pi Kr_1r_2})}=\frac{4\pi Kr_1r_2(\theta_2-\theta_1)}{(r_2-r_1)}$



Sol5. In SHM sum of kinetic and potential energy will be constant and average kinetic energy & average potential energy in one time will be remains same.

Sol6. Since 5Ω is connected across conductor so we can remove it.

$R_{eq}=1\Omega$

Sol7. Using truth table

A	B	V_0
0	0	0
0	1	1
1	0	1
1	1	1

Sol8. Load of mass will be equally distributed among the four columns so force on each columns will be 125×10^3N .

Cross section area of the column= $\pi[(1)^2-(0.5)^2]=2.355m^2$

Using young's modulus: $\varepsilon=\frac{\sigma}{Y}=\frac{F}{AY}=\frac{125\times10^3}{2.355\times2\times10^{11}}=2.65\times10^{-7}$

Sol9. Using $F=ma=m\frac{V}{T}$

$\Rightarrow m=FTV^{-1}$

Sol10. Force on q_1 :

$$F_{q_1}=q(V\times B)=4\pi(0.5c\hat{i})\times\frac{B_0}{\sqrt{2}}(\cos(kz-\omega t)\hat{i}+\cos(kz-wt)\hat{j})$$

$$=4\pi(0.5c)(\frac{B_0}{\sqrt{2}})\cos(k\frac{\pi}{k})\hat{k}$$

Force on q_2:

$$F_{q_2}=q(V\times B)=2\pi(0.5c\hat{i})\times\frac{B_0}{\sqrt{2}}(\cos(kz-\omega t)\hat{i}+\cos(kz-wt)\hat{j})$$

$$=2\pi(0.5c)(\frac{B_0}{\sqrt{2}})\cos(k\frac{3\pi}{k})\hat{k}$$

So ration will be 2:1

Sol11. Using Ideal gas Equation:

PV=nRT

$$\Rightarrow n=\frac{PV}{RT}=\frac{400\times10^3\times500\times10^{-6}}{8.3\times100}=0.008$$

$$\Rightarrow\frac{m_1}{2}+\frac{m_2}{32}=0.08$$

$$\Rightarrow16m_1+m_2=2.56$$

$$\Rightarrow16m_1+0.76-m_1=2.56$$

$$\Rightarrow m_1=0.12,m_2=0.64$$

$$\Rightarrow\frac{m_2}{m_1}=\frac{16}{3}$$

Sol12. For equilibrium net force acting on the system should be zero.

Sol13. Energy of electron in first excited state will be -3.4eV.

So total energy difference will be (2.6+3.4)eV.

Wavelength(λ)=$\frac{1242eV-nm}{6eV}$ =207 nm

Frequency=$\frac{c}{\lambda}=\frac{3\times10^8}{207\times10^{-9}}=1.45\times10^9$MHz

Sol14. Using conservation of linear momentum:

$$\Rightarrow 40\times 3m = 60\times m + v\times 2m$$

$$\Rightarrow v = 30 m/s$$

$$KE_i = \frac{1}{2}\times 3m\times (40)^2$$

$$KE_f = \frac{1}{2}\times m\times (60)^2 + \frac{1}{2}\times 2m\times (30)^2$$

$$\Rightarrow \frac{KE_f}{KE_i} = \frac{54}{48}$$

Fractional change in kinetic energy $= 1-\frac{KE_f}{KE_i} = \frac{1}{8}$

Sol15. $\vec{x} = |\vec{P}+\vec{Q}| = \sqrt{P^2+Q^2}$

$\vec{y} = |\vec{P}-\vec{Q}| = \sqrt{P^2+Q^2}$

$\Rightarrow |\vec{x}+\vec{y}| = \sqrt{x^2+y^2+2xy}$

$\Rightarrow \sqrt{3(P^2+Q^2)} = \sqrt{(\sqrt{P^2+Q^2})^2+(\sqrt{P^2+Q^2})^2+2(P^2+Q^2)\cos\theta_1}$

$\Rightarrow \theta_1 = 60^0$

Using same formula: $\theta_2 = 90^0$

Sol16. Time period $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

$$\Rightarrow T_i = T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow T_f = T' = 2\pi\sqrt{\frac{4l/3}{g(1-\frac{\rho}{\sigma})}} = 2\pi\sqrt{\frac{16l}{9g}}$$

$$\Rightarrow T' = \frac{4}{3}T$$

Sol17. Using faraday's law magnetic field should be outward and decreasing with time

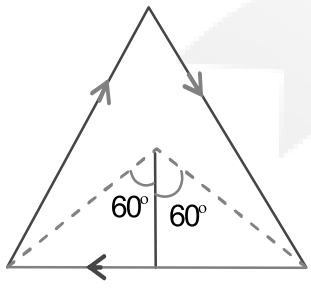
Sol18. Capacitor makes potential difference constant.

Sol19. using magnetic field due to straight wire:

$$B = \frac{\mu_0 i}{4\pi r}(\sin\alpha+\sin\beta)$$

$$= \frac{10^{-7}\times 1.5}{(\frac{0.09}{2\sqrt{3}})}\times(\sin 60^0+\sin 60^0) = 10^{-5}T$$

So, Magnetic field due to three wires $= 3\times 10^{-5}T$ inside the plane



Sol20. when electric field is parallel then it would provide zero flux.

SECTION - B

Sol1. Equation from the given graph:

$V^2 = 2x + 20$

$V^2 = 2x + 20$

$\Rightarrow 2V\dfrac{dV}{dt} = 2V$

$\Rightarrow a = 1m/s^2$

Sol2. From question:

9MSD=10VSD

$LC = 1MSD - 1VSD = 1MSD - \dfrac{9}{10}MSD = 0.1MSD = 0.1\times 1mm = 0.1mm$

Reading=10mm+8x0.1mm=10.8mm

Final reading=10.8mm-0.04mm=10.4mm=104×10^{-2}cm

So radius=52×10^{-2}cm

Sol3. Using Heat equation: $H = i^2Rt$

$\Rightarrow 192 = (4)^2R(1)$

$H = (8)^2R(5)$

$\Rightarrow H = 3840J$

Sol4. Signal bandwidth $= 12KHz$

No. of station $= \dfrac{6MHz}{12kHz} = 500$

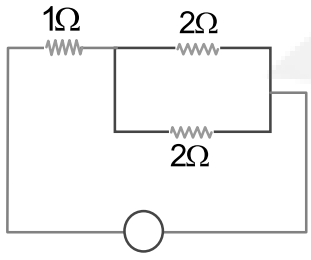
Sol5. $\dfrac{\Delta M}{M} = \dfrac{\Delta\mu}{\mu} = \dfrac{250}{500} = \dfrac{1}{2}$

Sol6. Distance of 4th bright fringe from central maxima $= \dfrac{4\lambda D}{d}$

So, $\dfrac{2.4}{100} = \dfrac{8\lambda D}{d} \Rightarrow \lambda = \dfrac{2.4\times 0.3\times 10^{-3}}{100\times 8\times 1.5} = 6\times 10^{-7}m$

$f = \dfrac{c}{\lambda} = \dfrac{3\times 10^8}{6\times 10^{-7}} = 5\times 10^{14}Hz$

Sol7. $X_c = \dfrac{1}{\omega c}, X_L = \omega L$ so at very high frequencies capacitor behaves as conductor and inductor behaves as open circuit. The effective impedance will be 2Ω.



Sol8. from given information: $i = A = 60^0, r = \dfrac{A}{2} = 30^0 \Rightarrow \mu = \sqrt{3} = \dfrac{c}{v}$

$$\Rightarrow v = \frac{c}{\sqrt{3}} \Rightarrow \text{time} = \frac{(\frac{5\sqrt{3}}{100})}{(\frac{3\times10^8}{\sqrt{3}})} = 5\times10^{-10}\,s$$

Sol9. Battery is connected while dielectric is inserted so potential difference will be remains same.

$$U_i = \frac{1}{2}cV^2$$

$$U_f = \frac{1}{2}KcV^2$$

$$\Rightarrow \Delta U = \frac{1}{2}(K-1)cV^2 = \frac{1}{2}\times1\times200\times10^{-9}\times200^2 = 4$$

Sol10. Adiabatic equation: $P_1V^{\gamma} = P_2V^{\gamma} \Rightarrow (200)(1200)^{\gamma} = P(300)^{\gamma}$

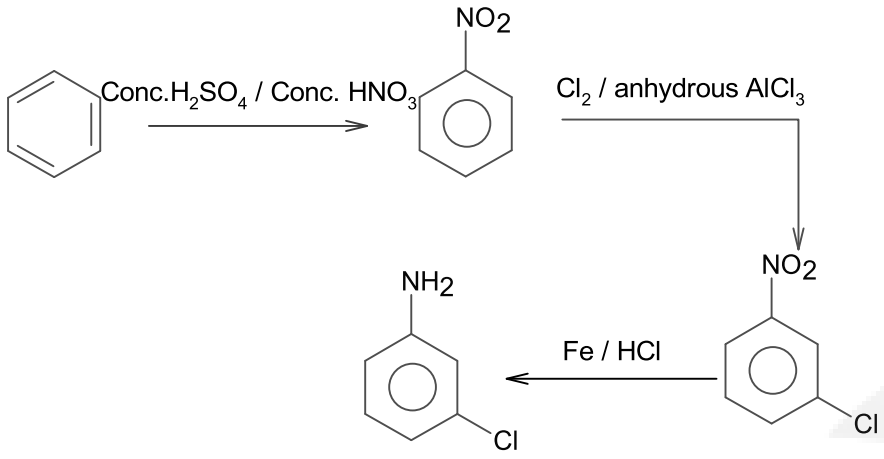
$$\Rightarrow P = (200)(4)^{\frac{3}{2}} = 1600KPa$$

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{240-480}{1.5-1} = -480J$$

PART – B (CHEMISTRY)

SECTION - A

Sol1.



Sol2. (a) Cell constant = $\left(\frac{\ell}{A}\right) = m/m^2 = m^{-1}$

(b) molar conductivity $(\lambda_m) = \frac{\kappa\times1000}{\text{molarity}}$

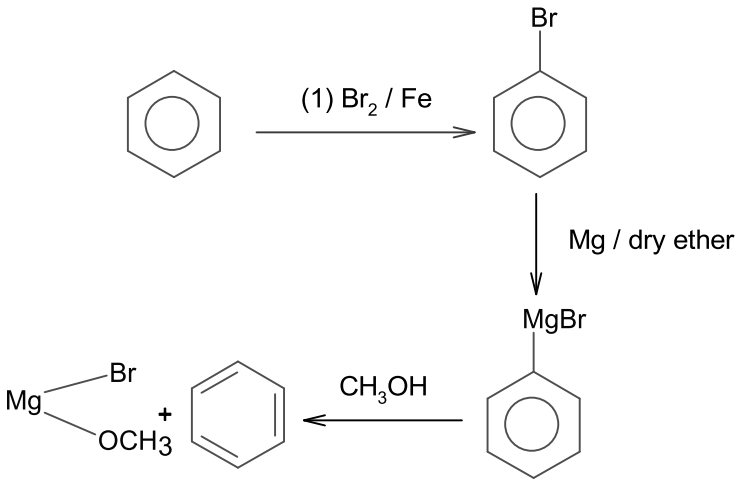
$= Scm^2mol^{-1}$

(c) Conductivity $(\kappa) = \frac{1}{\rho} = \frac{\ell}{RA}$

$= \Omega^{-1}m^{-1}$

(d) degree of dissociation = $\frac{\text{number of mole dissociated}}{\text{Total mole}}$, it is dimensionless.

Sol3.



Sol4. Stability order of oxides (X_2O) is,
$I_2O > Cl_2O > Br_2O$
Bonds of halogen & oxygen are covalent due to less EN difference.
Stability of (I –O) bond is higher due to less polarity and that of (Cl–O) bond is higher due to multiple bonding.

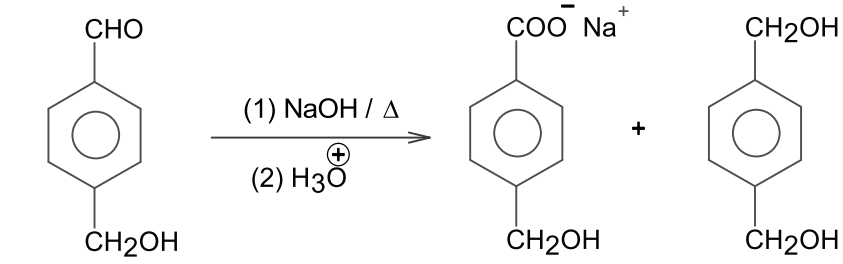
Sol5. Structure (I) is anti conformer.
Structure (II) is fully eclipsed conformer.
Structure (III) is skew or gauche conformer.
Structure(IV) is partially eclipsed.
∴ order of stability
(I) > (III) > (IV) > (II)
Order of P.E is (II) > (IV) > (III) > (I).

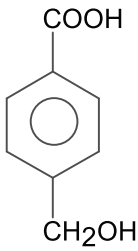
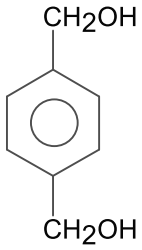
Sol6. $Eu(Z=63) = [_{54}Xe]4f^7 6s^2$

$Eu^{+2}(Z=63) = [_{54}Xe]4f^7$

Eu^{2+} in spite of having $4f^7$ configuration is strong reducing agent as it changes to most common oxidation state which is +3. Also, $E^0_{Eu^{3+}/Eu}$ is low that is $-2V$.

Sol7.



Products are,  and 

Sol8. Albumin is not an example of fibrous protein but globular protein.

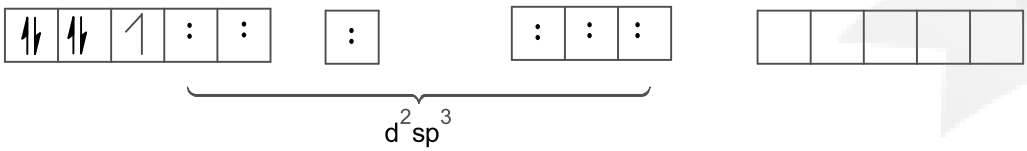
Sol9. $\left[Fe(CO)_4(C_2O_4)\right]^+$

$x + 4\times 0 - 2\times 1 = +1$

$x = +3$

$Fe^{+3} = 4s^0 3d^5$

In presence of strong ligand, i.e, CO.



Spin only magnetic moment

$= \sqrt{n(n+2)}B.M = \sqrt{1(1+2)}B.M$

$= \sqrt{3}B.M = 1.73B.M$

Sol10. (a) $\Delta G = \Delta G^\circ + RT\ell nQ$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ .....(1)

At equilibrium $\Delta G = 0,\ Q = K_{eq}$

$\therefore\ \Delta G^{o} = -RT\ell nK_{eq}$ .....(2)

$\therefore\ \Delta H^{o} - T\Delta S^{o} = -RT\ell nK_{eq}$

$$\ell nKeq = -\left(\frac{\Delta H^{o} - T\Delta S^{o}}{RT}\right)$$

(b) $W_{rev} = -nRT\ell n\left(\frac{V_f}{V_i}\right)$

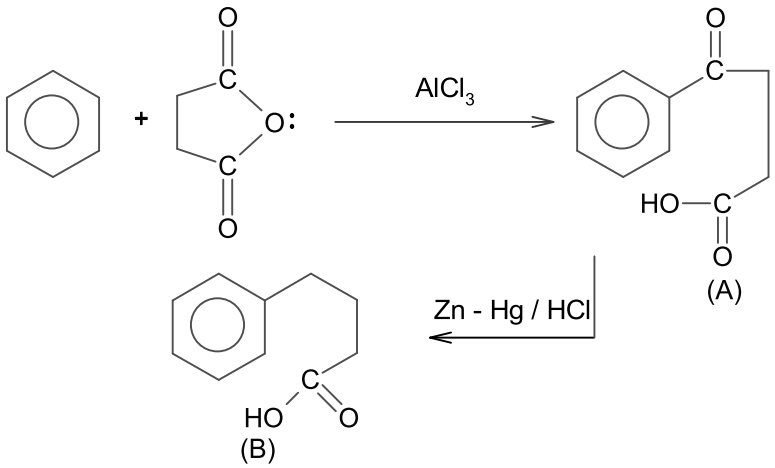
(c) $\Delta G = -T\Delta S_{Total}$ (at constant P)

$$\therefore\ \frac{\Delta G}{\Delta S_{Total}} = -T$$

(d) $\Delta G^{o} = -RT\ell nKeq$

$\therefore\ Keq = e^{\left(-\Delta G^{o}/RT\right)}$

Sol11.



Sol12. Glyptal, Dacron & PHBV are polyesters.
Novalac is copolymer of phenol & formaldehyde but not polyesters.

Sol13. $Mn^{+2} \longrightarrow$ group IV $\left(Mn^{+2}, Co^{+2}, Zn^{+2}, Ni^{2+}\right)$

$As^{+3} \longrightarrow$ group IIB $\left(As^{+3}, As^{+5}, Sb^{+3}, Sb^{+5}, Sn^{+2}, Sn^{+4}\right)$

$Cu^{+2} \longrightarrow$ group IIA $\left(Cu^{+2}, Pb^{+2}, Hg^{+2}, Cd^{+2}, Bi^{+3}\right)$

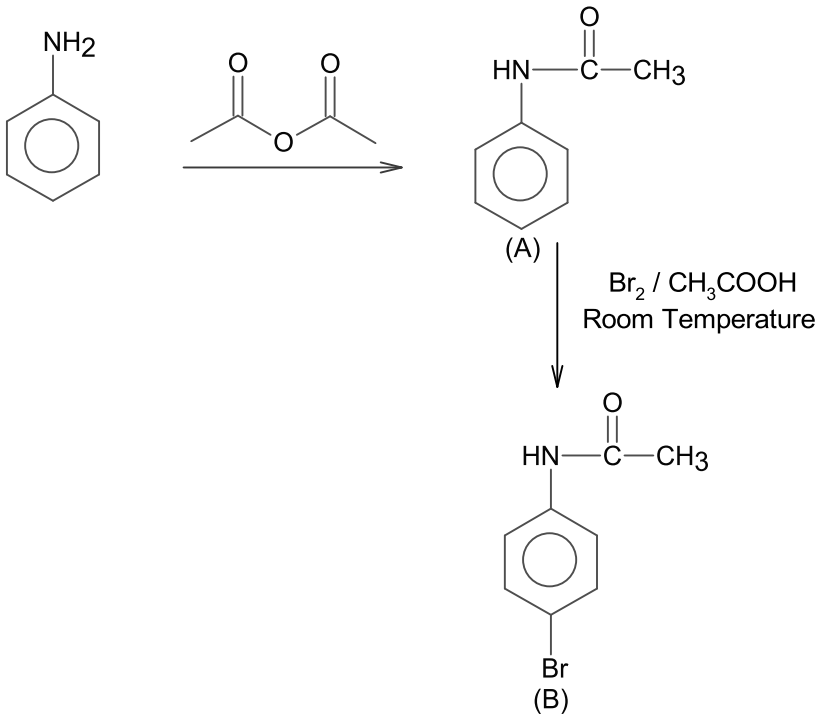
$A\ell^{+3} \longrightarrow$ group III $\left(Fe^{+3}, A\ell^{+3}, Cr^{+3}\right)$

Sol14. Ammonium salt in rain drop resulting wet deposition

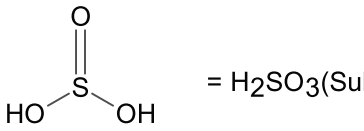
$NH_4^{+} - Salt + H_2O \longrightarrow NH_4OH$

Oxides of N & S settle down on ground as dry deposition (SO_2).

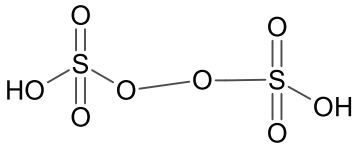
Sol15.



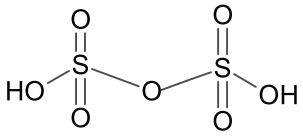
Sol16.



$= H_2SO_3$(Sulphurous acid)



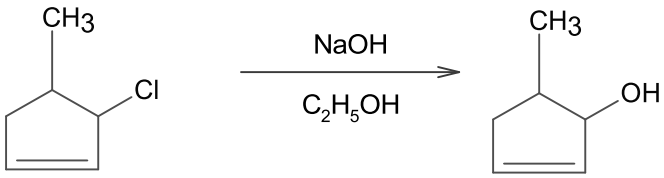
$= H_2S_2O_8$(Peroxodisulphuric acid)



$= H_2S_2O_7$(Pyrosulphuric acid)

Number of S = O bond are1,4,4.

Sol17.



Sol18. $H_2 \xrightarrow[hv]{high\ T} 2H$

$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

$NaOH_{(aq)} + Zn \longrightarrow Na_2ZnO_2 + H_2$

$H_2 \xrightarrow{2000K} 2H$

It is nearly 0.081%.

Sol19. ClO_2^-, Cl_2, Mn^{3+} can show disproportionation reaction while MnO_4^- cannot show disproportionation reaction is Mn is in +7 oxidation state.

Sol20. Lithium salts are extensively hydrated due to high hydration enthalpy of Li^+

$Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$ (order of polarizing power)

SECTION – B

Sol1. $C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \longrightarrow xCO_2 + \frac{y}{2}H_2O + \frac{Z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$

$C_2H_7N_1 + \left(2\times 2 + \frac{7}{2}\right)CuO \longrightarrow 2CO_2 + \frac{7}{2}H_2O + \frac{1}{2}N_2 + \left(2\times 2 + \frac{7}{2}\right)Cu$

$\therefore\ y = 7$

Ans. = 7

Sol2. $\left(\frac{x}{m}\right) = KP^{1/n}$

$\frac{10}{1} = K(100)^{1/n}$ .....(1)

$\frac{15}{1} = K(200)^{1/n}$ .....(2)

$\frac{V}{1} = K(300)^{1/n}$ .....(3)

Dividing (2) by (1)

$\frac{15}{10} = \left(\frac{2}{1}\right)^{1/n} = (2)^{1/n}$

$\frac{3}{2} = (2)^{1/n}$

$\log 3 - \log 2 = \frac{1}{n}\log 2$

$\frac{1}{n} = \frac{0.477 - 0.301}{0.301} = 0.5847 \approx 0.585$

Dividing (3) by (1)

$\frac{V}{10} = (3)^{1/n}$

$\log\left(\frac{V}{10}\right) = \frac{1}{n}\log 3$

$\log\left(\frac{V}{10}\right) = 0.585\times 0.477$

$= 0.279$

$\frac{V}{10} = 10^{0.279}$

$V = 10\times 10^{0.279} = 10^{1.279} = 10^x$

$\therefore\ x = 1.279 = 127.91\times 10^{-2} \approx 128\times 10^{-2}$

Ans. = 128

Sol3. $O_2^{-2} = 8\times 2 + 2 = 18e^-$

$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2pz}^2\pi_{2px}^2 = \pi_{2py}^2\pi_{2px}^{*2} = \pi_{2py}^{*2}$

Number of unpaired $e^- = 0$

Ans.= 0

Sol4. 50 ml of 1(M) HCl + 30 ml of 1(M) NaOH

$$\underset{30\times 1\text{mmol}}{NaOH} + \underset{50\times 1\text{mmol}}{HCl} \longrightarrow NaCl + H_2O$$

$$\underset{0\text{mmol}}{\quad} \qquad \underset{20\text{mmol}}{\quad}$$

$$\left[H^+\right]_{mix} = \frac{20}{50+30}M = \frac{20}{80}M = \frac{1}{4}M = 0.25M$$

$$pH = -\log\left[H^+\right] = -\log\left(2.5\times 10^{-1}\right)$$

$= 1 - 0.3979 = 0.60209$

$x\times 10^{-4} = 6021\times 10^{-4}$

$\therefore\ x = 6021$

Ans. = 6021

Sol5. Effective number of atom in C.C.P

$$\therefore = \frac{1}{8}\times 8 + \frac{1}{2}\times 6$$

= 4

Number of octahedral void = 4

Number of cations = 4

Number of anion = 4

Formula of compound = A_4B_4

Empirical formula = AB

Ans. x = 1.

Sol6. $\Delta T_b = iK_bm$

$$m = \frac{w\times 1000}{M\times W_{solvent}}$$

For acetone solution,

$$0.17 = 1\times 1.7\times \frac{1.22\times 1000}{M\times 100}$$

$$\frac{0.17}{1.7} = \frac{12.2}{M}$$

$$M = \frac{20.74}{0.17} = 122g/mole$$

For Benzene solution,

$2Acid \longrightarrow (Acid)_2 \qquad \therefore\ i = 1/2$

$\Delta T_b = i\times K_b\times m$

$$= \frac{1}{2}\times 2.6\times \frac{1.22\times 1000}{122\times 100}\,^\circ C$$

$= 0.13^\circ C = 13\times 10^{-2}\,^\circ C$

$\therefore\ x\times 10^{-2} = 13\times 10^{-2}$

$\therefore\ x = 13$

Sol7. $\log k = 20.35 - \frac{\left(2.47\times10^3\right)}{T}$

Comparing with,

$\log k = \log A - \frac{Ea}{2.303RT}$

$\frac{Ea}{2.303R} = 2.47\times10^3$

$Ea = \frac{2.47\times10^3\times2.303\times8.314}{1000}$ kJ / mole

= 47.29kJ / mole

Ans.= 47

Sol8. $Zn(30) = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10}$

$Zn^+ = [Ar]4s^1 3d^{10}$

Outermost electron is 4s electron,

$n = 4, \ell = 0, m = 0, s = \pm1/2$

Sol9. $Na_2O + H_2O \longrightarrow 2NaOH$

$w = 20g$

$V_{H_2O} = 500mL$

Mole of $Na_2O = \frac{20}{62}$

$\because$ 1mole of Na_2O gives 2 mole of NaOH

$\therefore$ $\left(\frac{20}{62}\right)$ mole of Na_2O gives $\left(2\times\frac{20}{62}\right)$ moles of NaOH

= $\frac{20}{31}$mole

Molarity of NaOH solution

= $\frac{\frac{20}{31}\times1000}{500}M = \frac{20}{31}\times2M = \frac{40}{31}M$

$= 1.29M$

$= 12.9\times10^{-1}M \approx 13\times10^{-1}M$

Ans. = 13

Sol10. Impurities present in electrolytic refining of blister Cu, removed as anode mud = Sb, Se, Te, Ag, Au, Pt

Ans. = 6

PART – C (MATHEMATICS)

SECTION – A

Sol1. $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$

Domain of $\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)$ is

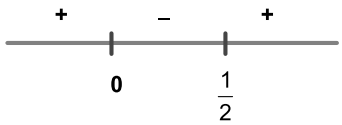
$-1\le\frac{3x^2+x-1}{(x-1)^2}\le 1$

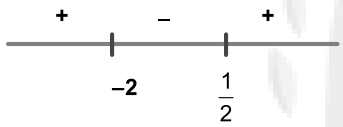
$-x^2-1+2x\le 3x^2+x-1$ and $3x^2+x-1\le x^2+1-2x$

$x\left(x-\frac{1}{4}\right)>0$

$(2x-1)(x+2)\le 0$

$\left(x-\frac{1}{2}\right)(x+2)\le 0$





$x\in\left[-\infty,0\right]\cup\left[\frac{1}{4},\infty\right)$

$x\in\left[-2,\frac{1}{2}\right]$

$x\in\left[-2,0\right]\cup\left[\frac{1}{4},\frac{1}{2}\right]$ .....(i)

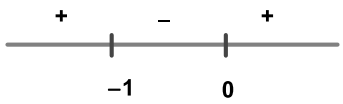
Domain of $\cos^{-1}\left(\frac{x-1}{x+1}\right)$

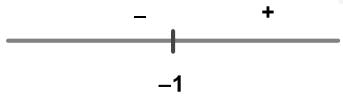
$-1\le\frac{x-1}{x+1}\le 1$

$\frac{x-1}{x+1}\ge -1$ and $\frac{x-1}{x+1}-1\le 0$

$\frac{2x}{x+1}\ge 0$

$\frac{2}{x+1}\ge 0$





$x\in\left(-\infty,-1\right]\cup\left[0,\infty\right)$

$x\in\left[-1,\infty\right)$

$x\in\left[0,\infty\right)$ .....(ii)

Domain of the function is $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$

Sol2. $y.\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(y^2/x^2\right)}{\phi'\left(y^2/x^2\right)}\right],x>0,\phi>0$

Let $\frac{y}{x}=t$

$\frac{dy}{dx}=t+x.\frac{dt}{dx}$

$$\frac{y}{x}.\frac{dy}{dx}=\left[\left(\frac{y}{x}\right)^2+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$$

$$t\left(t+x\frac{dt}{dx}\right)=\left[(t)^2+\frac{\phi(t^2)}{\phi'(t^2)}\right]$$

$$t^2+xt.\frac{dt}{dx}=t^2+\frac{\phi(t^2)}{\phi'(t^2)}$$

$$x.t.\frac{dt}{dx}=\frac{\phi(t^2)}{\phi'(t^2)}$$

$$\frac{t.\phi'(t^2)}{\phi(t^2)}.dt=\frac{1}{x}dx$$

$$\phi(t^2)=u$$

$$\phi'(t^2)2t=\frac{du}{dt}$$

$$t.\phi'(t^2)dt=\frac{du}{2}$$

Now, $\frac{1}{2}\int\frac{1}{u}du=\int\frac{1}{x}dx$

$$\Rightarrow\frac{1}{2}\ln(u)=\ln(x)+C$$

$$\Rightarrow\frac{1}{2}\ln(\phi(t^2))-\ln(x)+C$$

$$\Rightarrow\frac{1}{2}\ln\left(\phi\left(\frac{y}{x}\right)^2\right)=\ln x+C$$

At $x=1,\quad y=-1$

$$\frac{1}{2}\ln(\phi(1))=\ln(1)+C$$

$$C=\frac{1}{2}\ln(\phi(1))$$

$$\frac{1}{2}\ln\left(\phi\left(\frac{y}{2}\right)^2\right)=\ln x+\frac{1}{2}\ln(\phi(1))$$

$$\ln\left(\phi\left(\frac{y}{x}\right)^2\right)=\ln x^2+\ln(\phi(1))$$

$$x=2$$

$$\ln\left(\phi\left(\frac{y^2}{4}\right)\right)=\ln 4+\ln(\phi(1)$$

$$=\ln 4(\phi(1))$$

$$\phi\left(\frac{y^2}{4}\right)=4.\phi(1)$$

Sol3. $f(m+n)=f(m)+f(n)$

put $m=n=1, f(2)=f(1)+f(1)$

again put $m=2, n=1, f(3)=f(2)+f(1)$

and put $m=3, n=3, f(3+3)=f(3)+f(3), 2f(3)=f(6)=18 \Rightarrow f(3)=9$

$f(3)=3f(1)$

$f(1)=3, f(2)=6$

$f(2).f(3)=54$

Sol4. $32^{\tan^2 x}+32^{\sec^2 x}=81$

$32^{\tan^2 x}+32^{1+\tan^2 x}=81$

$33.32^{\tan^2 x}=81$

$32^{\tan^2 x}=\dfrac{81}{33}$

for $x \in [0, \pi/4]$ $\tan^2 x \in [0,1]$

One solution

Sol5. $\alpha=\lim\limits_{x\to\pi/4}\dfrac{\tan^3 x-\tan x}{\cos(x+\pi/4)}$

$=\lim\limits_{x\to\pi/4}\dfrac{3\tan^2 x.\sec^2 x-\sec^2 x}{-\sin(x+\pi/4)}$

$\alpha=-4$

$\beta=\lim\limits_{x\to 0}(\cos x)^{\cot x}$

$\beta=e^{\lim\limits_{x\to 0}\left(\frac{\cos x-1}{\tan x}\right)}$

$\beta=e^{\lim\limits_{x\to 0}\frac{-\sin x}{\sec^2 x}}$

$\beta=e^0=1$

Equation whose roots are α and β

$x^2+3x-4=0$

$a=1, b=3$

Sol6. $\vec{a}\times\left[(\vec{r}-\vec{b})\times\vec{a}\right]+\vec{b}\times\left[(\vec{r}-\vec{c})\times\vec{b}\right]+\vec{c}\times\left[(\vec{r}-\vec{a})\times\vec{c}\right]=\vec{0}$

$=(\vec{a}-\vec{a})(\vec{r}-\vec{b})-\left(\vec{a}(\vec{r}-\vec{b})\right)\vec{a}+(\vec{b}.\vec{b})(\vec{r}-\vec{c})+\left(\vec{b}(\vec{r}-\vec{c})\right)\vec{b}+(\vec{c}.\vec{c})(\vec{r}.\vec{a})+\left(\vec{c}.(\vec{r}-\vec{a})\right)\vec{c}$

As $|\vec{a}|^2=|\vec{b}|^2=|\vec{c}|^2$

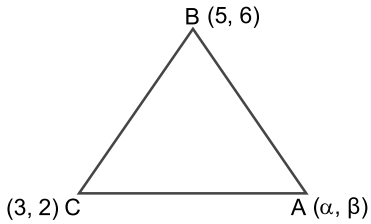
$=|\vec{a}|^2\left(3\vec{r}-(\vec{a}+\vec{b}+\vec{c})\right)-\left((\vec{a}.\vec{r})\vec{a}+(\vec{a}.\vec{r})\vec{b}+(\vec{c}.\vec{r})\vec{c}\right)$

Let $\vec{r}=x\vec{a}+y\vec{b}+z\vec{c}$

$=|\vec{a}|^2\left(2\vec{r}-(\vec{a}+\vec{b}+\vec{c})\right)$

$\vec{r}=\dfrac{\vec{a}+\vec{b}+\vec{c}}{2}$

Sol7. $\begin{vmatrix} \alpha & \beta & 1 \\ 5 & 6 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 24$



$4\alpha - 2\beta - 8 = \pm 24$

$4\alpha - 2\beta = 24 + 8$, $4\alpha - 2\beta = -24 + 8$

$2(2\alpha - \beta) = 32$ $2\alpha - \beta = -8$

Distance from origin

$D = \sqrt{\alpha^2 + (2\alpha + 8)^2}$

$D^2 = 5\alpha^2 + 32\alpha + 64$

$\frac{d(D^2)}{d\alpha} = 10\alpha + 32$

$10\alpha + 32 = 0$

$\alpha = -\frac{16}{5}$

$D = \sqrt{\left(\frac{-16}{5}\right)^2 + \left(\frac{8}{5}\right)^2}$

$D = \frac{8}{\sqrt{5}}$

If $2\alpha - \beta = 16$

$D = \frac{16}{\sqrt{5}}$

Sol8. Equation of intersection of line

$\frac{x-0}{1} = y = \frac{z-0}{-1}$

Let $\vec{r} = \hat{i} - \hat{k}$

Direction ratio of $\overrightarrow{PQ}$

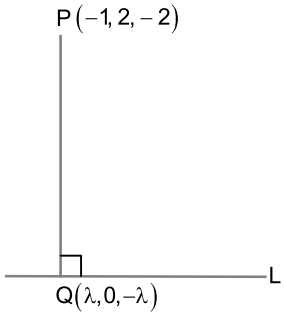
$= (\lambda + 1, -2, 2 - \lambda)$

$\overrightarrow{PQ} \perp \vec{r}$

$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$

$\lambda = \frac{1}{2} \Rightarrow Q\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$PQ = \frac{\sqrt{34}}{2}$



Sol9. $b^2x^2 + a^2y^2 = a^2b^2$

$\left(b^2x^2 + a^2(ab - x^2)\right) = a^2b^2$

$x^2 = \frac{ba^2(b-a)}{b^2 - a^2}$, $y^2 = \frac{ab^2}{a+b}$

Point of intersection

$$\left(a\sqrt{\frac{b}{a+b}}, b\sqrt{\frac{a}{a+b}}\right)$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{2x}{a^2}+\frac{2y}{b^2}.\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-b^2x}{a^2y}\rightarrow m_1$$

$$x^2+y^2=ab$$

$$2x+2y.\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{y}\rightarrow m_2$$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1.m_2}\right|$$

$$=\left|\frac{\frac{-b^2x}{a^2y}+\frac{x}{y}}{1+\frac{b^2x^2}{a^2y^2}}\right|$$

$$=\left|\frac{a-b}{\sqrt{ab}}\right|$$

Sol10. Total ways = 6!
Ways satisfying g(3)= 2g(1) is 3
Number of onto function 3 x 4!
Probability = $\frac{1}{10}$

Sol11. As $\sim(A\Rightarrow B)=A\wedge\sim B$

$$\sim((pvr)\Rightarrow(qvr))$$

$$=(pvr)\wedge(\sim r\wedge\sim q)$$

$$=((pvr)\wedge\sim r)\wedge(\sim q)$$

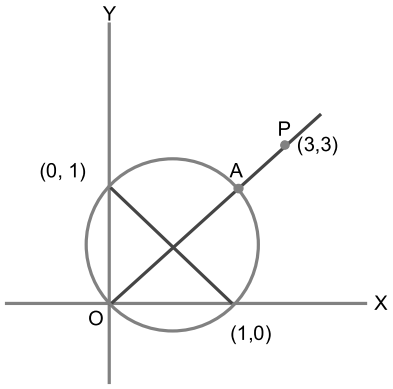
$$=(p\wedge\sim r)\vee(r\wedge\sim r)\wedge(\sim q)$$

$$=(p\wedge\sim r)$$

Sol12. Minimum distance AP = OP – OA

$= 3\sqrt{2} - \sqrt{2}$

$= 2\sqrt{2}$



Sol13. $\alpha + \beta + \gamma = 2\pi$

$$\Delta = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2\cos\alpha.\cos\beta.\cos\gamma$

$= \sin^2\alpha - \cos^2\beta - \cos\gamma(\cos\gamma - 2\cos\alpha.\cos\beta)$

$= -\cos(\alpha+\beta)\cos(\alpha-\beta) - \cos\gamma(\cos(\alpha+\beta) - 2\cos\alpha\cos\beta)$

$= -\cos\gamma\cos(\alpha-\beta) + \cos\gamma\cos(\alpha-\beta) = 0$

Sol14. $\frac{S_{10}}{S_P} = \frac{100}{P^2}$

$$S_P = \frac{S_{10}.P^2}{100} \Rightarrow S_{11} = \frac{S_{10}.121}{100}$$

$$\frac{a_{11}}{a_{10}} = \frac{S_{11} - S_{10}}{S_{10} - S_9} = \frac{S_{10}.\frac{121}{100} - S_{10}}{S_{10} - \frac{S_{10}.81}{100}}$$

$$= \frac{21}{19}$$

Sol15. $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$$x + 1 + \log_2\left[\frac{10.2^x - 1}{(3+2^x)^2}\right] - x = 0$$

$$1 + \log_2\left[\frac{10.2^x - 1}{(3+2^x)^2}\right] = 0$$

$$\frac{10.2^x - 1}{9 + (2^x)^2 + 6.2^x} = \frac{1}{2}$$

$$(2^x)^2 - 14.2^x + 11 = 0$$

$2^x = y$

$$y^2 - 14y + 11 = 0$$

$$y = 7 \pm \frac{\sqrt{152}}{2}$$

$$2^{x1} = y_1 = 7 + \frac{\sqrt{152}}{2}, y_2 = 7 - \frac{\sqrt{152}}{2} = 2^{x2}$$

$$x_1 = \log_2\left(7 + \frac{\sqrt{152}}{2}\right)$$

$$x_2 = \log_2\left(7 - \frac{\sqrt{152}}{2}\right)$$

$$x_1 + x_2 = \log_2\left(49 - \frac{152}{4}\right)$$

$$x_1 + x_2 = \log_2 11$$

Sol16. $f(0) = 0, f(1) = 1, f(2) = 2$

$h(x) = f(x) - x$ has three roots

$\Rightarrow h'(x) = f'(x) - 1$, has at least two roots

$\Rightarrow h''(x) = f''(x)$ has at least one root.

Sol17. $$I = \pi^2 \int_0^2 \left(\sin\left(\frac{\pi x}{2}\right)\right)(x - [x])^{[x]}\, dx$$

$$I = \pi^2 \int_0^1 \sin\left(\frac{\pi x}{2}\right).x^o dx + \pi^2 \int_1^2 \sin\left(\frac{\pi x}{2}\right)(x-1)^1\, dx$$

$$I = \pi^2\left[\frac{-2}{\pi}\cos\left(\frac{\pi x}{2}\right)\right]_0^1 + \pi^2\left[(x-1)\frac{2}{\pi}\left(-\cos\frac{\pi x}{2}\right)\right]_1^2 - \int_1^2 \frac{2}{\pi}\left(-\cos\left(\frac{\pi x}{2}\right)dx\right)$$

$$I = \pi^2\left(\frac{2}{\pi}\right) + \frac{2\pi^2}{\pi}[1-0] + 2\pi \times \frac{2}{\pi}\left[\sin\frac{\pi x}{2}\right]_1^2$$

$$I = 2\pi + 2\pi + 4(0-1)$$

$$I = 4\pi - 4$$

Sol18. $$\frac{dy}{dx} = \frac{2^x y + 2^y.2^x}{2^x + 2^{x+y}\log_e 2}$$

$$\frac{dy}{dx} = \frac{2^x\left(y + 2^y\right)}{2^x\left(1 + 2^y \log_e 2\right)}$$

$$\int \frac{1 + 2^y \log_e 2}{y + 2^y} dy = \int dx$$

$$\Rightarrow \ln\left|y + 2^y\right| = x + c$$

$$y(0) = 0$$

$$\ln\left|y + 2^y\right| = x$$

$$y = 1$$

$$x = \ln 3$$

$x \in (1,2)$

Sol19. a, b, c, d, e be 5 unknown
n = 7, mean = 8, variance = 16
sum of observations = $7 \times 8 = 56$

mean of 5 remaining observation = $\frac{56-8-6}{25} = \frac{42}{5}$

$$16 = \frac{\sum x_i^2}{7} - 64$$

$$\sum x_i^2 = 560$$

$a^2 + b^2 + c^2 + d^2 + e^2 = 460$

$= \frac{460}{5} - \left(\frac{42}{5}\right)^2$

$= \frac{536}{25}$

Sol20. Let point on ellipse $(2\sin\theta,\ 3\cos\theta)$ and the mid point of line segment joining $(-3,-5)$ and $(2\sin\theta,\ 3\cos\theta)$ will be (h,k)

$\frac{2\sin\theta - 3}{2} = h \quad \frac{3\cos\theta - 5}{2} = k$

$2\sin\theta = 2h + 3 \quad 3\cos\theta = 2k + 5$

$\sin\theta = \frac{2h+3}{2} \quad \cos\theta = \frac{2k+5}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$

$36x^2 + 16y^2 + 108x + 80y + 145 = 0$

SECTION – B

Sol1. $(2,2,-2)$ lie in a plane

$\Rightarrow 2 + 6 + 4 + \beta = 0$

$\Rightarrow \beta = -12$

Line is perpendicular to normal

$\alpha(1) - 5(3) + 2(-2) = 0$

$\alpha = 19$

$\alpha + \beta = 7$

Sol2. $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f''(-1) = 0$

$-6a + 2b = 0$

$\Rightarrow b = 3a$

$f'(1) = 0$

$3a+6b+c=0$

$c=-99$

$f(1)=-10$

$-5a+d=-10$ ……..(i)

$f(-1)=6$

$11a+d=6$ …….(ii)

$a=1, d=-5, b=3, c=-9$

$f(x)=x^3+3x^2-9x-5$

$f(3)=27+27-27-5=22$

Sol3. $\int\frac{\sin x}{\sin^3 x+\cos^3 x}dx$

$I=\int\frac{\tan x\sec^2 x}{\tan^3 x+1}dx$

Let, $\tan x=t$

$\sec^2 x dx=dt$

$\int\frac{tdt}{t^3+1}=\int\frac{t}{(t+1)(t^2-t+1)}dt$

$\frac{t}{(t+1)(t^2-t+1)}=\frac{A}{(t+1)}+\frac{Bt+c}{t^2-t+1}$

$t=A(t^2-t+1)+(Bt+c)(t+1)$

$A=\frac{-1}{3}, B=\frac{1}{3}, C=\frac{1}{3}$

$I=\frac{-1}{3}\int\frac{1}{t+1}dt+\frac{1}{3}\int\frac{t+1}{t^2-t+1}dt$

$=\frac{-1}{3}\ln(\tan x+1)+\frac{1}{6}\ln(\tan^2 x-\tan x+1)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+c$

The value of $18(\alpha+\beta+\gamma^2)$ is equal to 3.

Sol4. $\frac{1}{2}\int_0^{3/2}(1+4x-x^2)dx$

$=\frac{1}{2}\left[x+2x^2-\frac{x^3}{3}\right]_0^{3/2}=\frac{39}{8}$

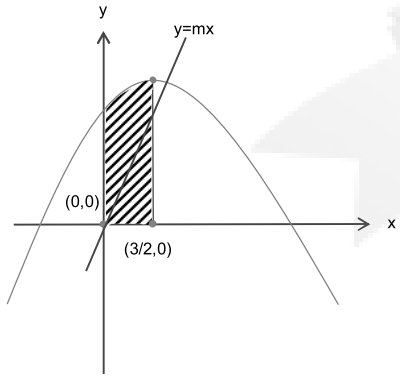
$\int_0^{3/2}mx=\frac{9m}{8}$

(as per question)

$\Rightarrow\frac{39}{8}=\frac{9m}{4}$

$m=\frac{39}{18}$

$12m=26$



Sol5. The tangent to the parabola

$y^2 = 8x$ at $(2,-4)$ is $-4y = 4(x+2)$

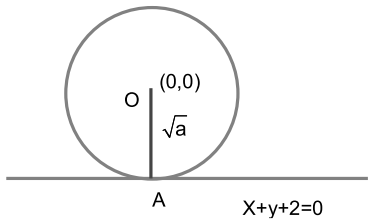
$x + y + 2 = 0$

$OA = \sqrt{a}$

$\Rightarrow \left|\frac{0+0+2}{\sqrt{2}}\right| = \sqrt{a}$

$\sqrt{2} = \sqrt{a}$

$a = 2$



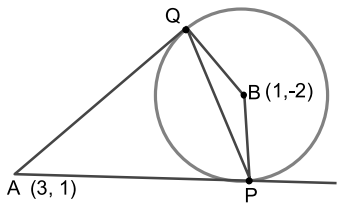
Sol6. $AP = \sqrt{9+1-6+4+1}$

$AP = 3 = AQ$

$r = \sqrt{1+4-1} = 2$

$\tan\theta = \frac{3}{2}$

$\frac{\text{Area of } \Delta APQ}{\text{Area of } \Delta BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$



Sol7. Total 4 digit number

9	10	10	10	= 9000

4 digit divisible by 7

1001, 1008, ------ 9996

$9996 = 1001 + (n_1 - 1)\,7$

$n_1 = 1286$

4 digit no divisible by 3

1002, 1005, --------9999

$9999 = 1002 + (n_2 - 1)3$

$n_2 = 3000$

4 digit number visible by 21

1008, 1031, ------ 9996

$n_3 = 429$

4 digit number divisible by 7 or 3

$= 9000 - 1286 - 3000 + 429$

$= 5143$

Sol8. $a^{10}b^{10}\left(\frac{1}{b} + \frac{2}{a} + 4\right)^{10}$

$a^{10}b^{10}\frac{10!\left(\frac{1}{b}\right)^{r_1}\left(\frac{2}{a}\right)^{r_2}.4^{10-r_1-r_2}}{r_1!r_2!(10-r_1-r_2)!}$

$r_1 = 2, r_2 = 3$

a^7b^8 is $\frac{10!2^3.4^{10-2-3}}{2!3!5!} = \frac{2^{13}.10!}{2!3!5!} = 2^{16}.315$

$k = 315$

Sol9. $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} \ldots\ldots\ldots\infty$

$$\frac{S}{5}=\frac{7}{5^2}+\frac{9}{5^2}+\frac{13}{5^4}+-------\infty$$

$$\frac{4S}{5}=\frac{7}{5}+\frac{2}{5^2}+\frac{4}{5^4}+\frac{6}{5^4}+----$$

$$\frac{4S}{5}-\frac{7}{5}=\frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+------$$

Let $\frac{4S-7}{5}=t$

$$\frac{4t}{5}=\frac{2}{25}\left\{\frac{1}{\left(1-\frac{1}{5}\right)^2}\right\}=\frac{1}{10}$$

$$t=\frac{1}{8}$$

$$\frac{4S-7}{5}=\frac{1}{8}$$

$$4S=\frac{5}{8}+7$$

$$4S=\frac{5+56}{8}$$

$$S=\frac{61}{32}$$

$\therefore 160S=305$

Sol10. $I-A^3-3A+3A^2=I-A^3$

$\Rightarrow 3A^2-3A=0$

$\Rightarrow 3A(A-I)=0$

$\Rightarrow A^2=A$

$$\begin{bmatrix}a^2 & ab+bd\\ 0 & d^2\end{bmatrix}=\begin{bmatrix}a & b\\ 0 & d\end{bmatrix}$$

$a^2=a$	$d^2=d$	$b(a+d)=b$
$a=0,1$	$d=0,1$	$a+d=1$

Total number of ways = 8